01 Aug 2006

Microwave Quantitative NDE Technique for Dielectric Slab Thickness Estimation using the MUSIC Algorithm

Mohamed A. Abou-Khousa

R. Zoughi
Missouri University of Science and Technology, zoughi@mst.edu

Recommended Citation

M. A. Abou-Khousa and R. Zoughi, "Microwave Quantitative NDE Technique for Dielectric Slab Thickness Estimation using the MUSIC Algorithm," *Proceedings of the Review of Progress in Quantitative Nondestructive Evaluation (2006, Portland, OR)*, vol. 894, pp. 440-447, American Institute of Physics (AIP), Aug 2006.
The definitive version is available at https://doi.org/10.1063/1.2718005

MICROWAVE QUANTITATIVE NDE TECHNIQUE FOR DIELECTRIC SLAB THICKNESS ESTIMATION USING THE MUSIC ALGORITHM

M.A. Abou-Khousa and R. Zoughi

Applied Microwave Nondestructive Testing Laboratory (*amntl*)
Electrical and Computer Engineering Department
University of Missouri-Rolla
Rolla, MO 65409

ABSTRACT. Non-invasive monitoring of dielectric slab thickness is of great interest in various industrial applications. This paper focuses on estimating the thickness of dielectric slabs, and consequently monitoring their variations, utilizing wideband microwave signals and the MUtiple SIgnal Characterization (MUSIC) algorithm. The performance of the proposed approach is assessed by validating simulation results with laboratory experiments. The results clearly indicate the utility of this overall approach for accurate dielectric slab thickness evaluation.

Keywords: dielectric slabs, microwaves, MUSIC algorithm, spectrum estimation, thickness estimation.
PACS: 81.05.Je, 81.70.Ex, 84.40.-x.

INTRODUCTION

Non-invasive monitoring of dielectric slab thickness is of great interest in various industrial applications. Examples of these applications include monitoring the thickness variations in ceramic, thermal barrier and synthetic rubber coatings, conveyed plastics and textiles, concrete pavements, and refractory walls. Here, we address the problem of estimating the thickness of dielectric slabs, and consequently monitoring their variations, utilizing wideband microwave signals and high-resolution spectrum estimation technique. This method is proposed as an alternative for accurate thickness evaluation, which requires either a very narrow pulse or a very wideband signal. This problem is further exasperated when the electrical thickness of the slab is small which requires narrower pulses or wider band signals. Neither of these two are generally possible at microwave frequencies for very thin slabs. Although the proposed method also requires a wideband signal, the required bandwidth is within one waveguide band, which is easily accommodated at microwave frequencies.

Basically, an antenna is used to illuminate the slab with a wideband microwave signal and intercept the reflected signal. Microwave signals are very sensitive to the boundaries at dielectric slab interfaces [1]. Therefore, the interrogating microwave signal is reflected toward the transmitter upon incidence on these boundaries. The reflected microwave signal is coherently detected and, subsequently processed using spectrum

CP894, *Review of Quantitative Nondestructive Evaluation Vol. 26*, ed. by D. O. Thompson and D. E. Chimenti

estimation techniques to determine the location of the slab interfaces, and hence the slab thickness. In this context, the spectrum estimation techniques are expected to be able to detect two reflections spaced by the time it takes the signal to travel between the interfaces (round trip). Thus, the spectrum estimators must be capable of resolving the time delay between the interfaces. In general, the resolution in estimating the delay time is proportional to the bandwidth of the transmitted signal (the higher the bandwidth, the higher the resolution).

Conventionally, Fast Fourier Transform (FFT)-based spectrum estimation is applied to the reflected signal in order to detect the slab interfaces. The thickness resolution as offered by the FFT-based spectrum estimation is fundamentally limited by the transmitted signal bandwidth [2]. The resolution limit for an FFT-based spectrum estimators is $\delta d \geq \text{phase velocity in the slab}/2\times\text{signal bandwidth}$. Consequently, relatively thin slabs constitute a challenge to such approach since a very narrow pulse or very wide sweep range is needed to estimate the thickness of such slabs. While using narrow pulses at the microwave frequencies requires prohibitively complex circuitry, extending the bandwidth beyond the waveguide (or a horn antenna) band is not readily available.

To overcome the abovementioned limitations, we propose using the MUtiple SIgnal Characterization (MUSIC) algorithm as spectrum estimator. The MUSIC algorithm offers significantly higher thickness resolution compared to the conventional FFT-based algorithms. It will be shown that the MUSIC algorithm can be applied to estimate the thickness of thin slabs using a bandwidth equal to that of a waveguide band (very simple hardware).

METHODOLOGY

A dielectric slab of thickness d_s located in free-space is illuminated by a wideband uniform plane electromagnetic wave as shown in FIGURE 1. The dielectric slab

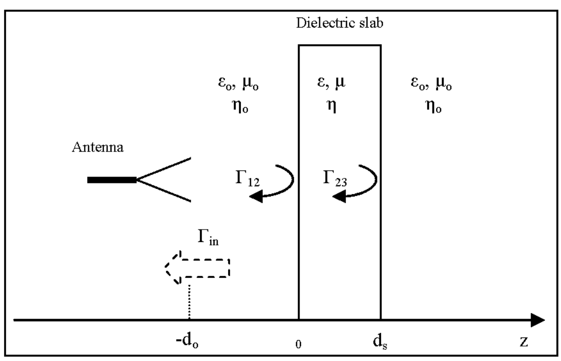


FIGURE 1: Schematic of the considered setup.

is assumed to have a complex permittivity $\varepsilon = \varepsilon_o(\varepsilon_r^{'} - j\varepsilon_r^{"})$ where $\varepsilon_o = 8.854 \times 10^{-12}$ [F/m] is the permittivity of free-space, $\varepsilon_r^{'}$ and $\varepsilon_r^{"}$ are the real and imaginary parts of the relative, to free-space, permittivity of the slab. While the real part describes the ability of the material to store the incident electric energy, the imaginary part is a measure of the material's ability to absorb energy. Finally, the slab is assumed to be made of non-magnetic material, i.e., the relative permeability, $\mu_r = 1$.

At each interface, the incident wave experiences a reflection due the change in the dielectric properties at the interface. These reflections are referred to as intrinsic reflections. The first reflection Γ_{12} takes place at the air-slab interface located at $z = 0$. The remaining portion of the wave propagates in the slab till it reaches the slab-air interface at $z = d_s$ where it undergoes another reflection Γ_{23}. Due to the intrinsic reflections, multiple reflections are created within the slab as well. In general, if the magnitudes of intrinsic reflections are small compared to unity, the multiple reflections can be ignored and the total reflection $\Gamma_{in}(f)$ measured at the input port of the antenna as a function of the wave frequency f can be approximated as [3]:

$$\Gamma_{in}(f) \approx \Gamma_{12}e^{-2j\beta(f)d_o} + \Gamma_{23}\alpha e^{-2j\beta(f)[d_o+\sqrt{\varepsilon_r^{'}}d_s]} \tag{1}$$

where the propagation constant in free-space is $\beta(f) = 2\pi f / c$ [rad/m], c is the speed of light in free-space, d_o is the distance from the antenna to the first slab interface, and $\alpha = e^{-bd_s}$, b is a positive attenuation parameter given as:

$$b = \mathrm{Re}\{2\pi f\sqrt{\varepsilon_r^{'} - j\varepsilon_r^{"}} / c\}, \quad \text{[Np/m]}$$

The intrinsic reflections are given by:

$$\Gamma_{12} = -\Gamma_{23} = \frac{\eta - \eta_o}{\eta + \eta_o} \tag{2}$$

where $\eta_o = 120\pi$ [Ω] is the intrinsic impedance of free-space, and $\eta = \eta_o / \sqrt{\varepsilon_r^{'} - j\varepsilon_r^{"}}$ is the intrinsic impedance of the slab.

In order to detect both interfaces, and hence determine the thickness, the frequency of the incident wave is swept from f_1 to f_L with sweep step size δ_f. At each frequency, the input reflection coefficient is measured and saved for post processing. Using (1) the measurement model is expressed as:

$$\Gamma_m(f_i) \approx \Gamma_{12}e^{-2j\beta(f_i)d_o} + \Gamma_{23}\alpha e^{-2j\beta(f_i)[d_o+\sqrt{\varepsilon_r^{'}}d_s]} + w_i \tag{3}$$

where Γ_m is the measured input reflection coefficient, f_i is the i^{th} frequency point, $i = 1, 2, ..., L$, and w_i is a sample of complex white noise process with zero mean and variance $N_0 / 2$ per dimension.

The measurement model of (3) describes a superposition of two (perfectly correlated) complex sinusoids in noisy background. The objective is to estimate the *electrical thickness*, $\sqrt{\varepsilon_r^{'}}d_s$ from this model. This can be accomplished by estimating the delay-domain spectrum, $S(d)$, of the observed sequence. Subsequently, the slab thickness can be estimated as $\hat{d}_s$ = Estimated Electrical Thickness/$\sqrt{\varepsilon_r^{'}}$. In the following, a procedure to estimate the electrical thickness from the measured signal using the MUSIC algorithm is described.

MUSIC SPECTRUM ESTIMATOR

The MUSIC algorithm introduced in [4] belongs to the high-resolution eigenstructure-based spectrum estimators. It has shown great promise in terms of the resolution it offers and the estimate bias characteristics especially at high signal-to-noise ratios (SNR). The MUSIC algorithm utilizes the eigenstructure of the measured signal's covariance matrix to construct two orthogonal subspaces; the signal subspace and the noise subspace. Thereafter, it searches for the signal parameters by projecting variable delay scanning vector into the noise subspace. If a certain scanning vector indeed belongs to one of the impinging signals, it essentially projects minimal power into the noise subspace; ideally zero. The delay-domain spectrum, $S(d)$, is estimated by measuring the projection power as a function of the delay used to construct the scanning vector.

The measurement model of (3) can be written compactly as:

$$\mathbf{s} = \mathbf{A\Gamma} + \mathbf{w} \tag{4}$$

where $\mathbf{s}$ is $L \times 1$ measurement vector given as $\mathbf{s} = \left[\Gamma_m(f_1) \quad \Gamma_m(f_2) \quad \cdots \quad \Gamma_m(f_L)\right]^T$, $\mathbf{A} = \left[\mathbf{a}(d_o) \quad \mathbf{a}(d_s)\right]$ is a $L \times 2$ delay scanning matrix whose columns are the delay scanning vectors of the form $\mathbf{a}(d) = \left[e^{-2j\beta(f_1)d} \quad e^{-2j\beta(f_2)d} \quad \cdots \quad e^{-2j\beta(f_L)d}\right]^T$, $\mathbf{\Gamma} = \left[\Gamma_{12} \quad \Gamma_{23}\right]^T$ is the 2×1 intrinsic reflections vector, and $\mathbf{w} = \left[w_1 \quad w_2 \quad \cdots \quad w_L\right]^T$ is the measurement noise vector.

The sample covariance matrix of the measurements is estimated from (4) as:

$$\hat{\mathbf{\Phi}} = E\{\mathbf{ss}^H\} = \mathbf{A\Gamma\Gamma}^H\mathbf{A}^H + N_0\mathbf{I} \tag{5}$$

In general, the MUSIC algorithm performs well in the case of uncorrelated bearings. Since the reflections are perfectly correlated, the covariance matrix of the measured signal $\hat{\mathbf{\Phi}}$ is singular, and consequently, the dimensionality of the signal subspace no longer equals to the number of reflections [5]. This situation has shown to be resolvable using measurement smoothing techniques at the expense of reducing the effective bandwidth (and hence the potential delay resolution). Measurement smoothing is basically a preprocessing scheme used to destroy the coherence of the measured signal while preserving the eigenstructure of the covariance matrix. Forward and backward smoothing (FBS) has shown to be one of the most effective pre-processing schemes used to achieve that end [6].

The MUSIC algorithm for a general case of correlated reflections is summarized in the following steps.

I. For L frequency points, construct q sub-measurements array each of p frequency points such that $L = q + p - 1$.
II. Compute the $p \times p$ smoothed sample covariance matrix as:

$$\hat{\mathbf{\Phi}}_s = \frac{1}{2q}\sum_{j=1}^{q} \hat{\mathbf{\Phi}}_{\mathbf{j}} + \mathbf{J}\hat{\mathbf{\Phi}}_{\mathbf{j}}^*\mathbf{J} \tag{6}$$

where, $\hat{\mathbf{\Phi}}_{\mathbf{j}} = \mathbf{s}_{\mathbf{j}}\mathbf{s}_{\mathbf{j}}^{H}$ is the sub-measurements array covariance matrix, $\mathbf{s}_{\mathbf{j}}$ is a $p \times 1$ vector containing the measured reflection coefficients at the frequency points of the j^{th} sub-measurement array, and the $p \times p$ transition matrix is:

$$J = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ \vdots & 0 & 1 & 0 \\ 0 & \iddots & 0 & \vdots \\ 1 & 0 & \cdots & 0 \end{bmatrix}$$

III. Apply the eigenvalue decomposition to the smoothed covariance matrix, $\mathbf{\Phi}_{\mathbf{s}}\mathbf{V} = \mathbf{D}\mathbf{V}$ where $\mathbf{D} = diag(\lambda_1, \lambda_2, \ldots, \lambda_p)$, λ_i being the i^{th} eigenvalue, such that $\lambda_1 \le \lambda_2 \le \ldots < \lambda_{p-1} \le \lambda_p$, and $\mathbf{V}$ is matrix whose columns are the corresponding eigenvectors.

IV. Using the Minimum Description Length criteria [7], estimate the number of reflections, N, as:

$$MDL(m) = -\log \left[\frac{\prod_{i=m+1}^{p} \lambda_i^{\frac{1}{(p-m)}}}{\frac{1}{(p-m)} \prod_{i=m+1}^{p} \lambda_i} \right]^{q(p-m)} + 0.5m(2p-m)\log(q)$$

$$N = \min(MDL)$$

V. With the above ordering of the eigenvalues, the eigenvectors corresponding to the noise subspace are the first $p-N$ columns of $\mathbf{V}$. Construct the noise subspace eigenvectors matrix $\mathbf{V}_w$ which has the first $p-N$ columns of $\mathbf{V}$.

VI. Compute MUSIC delay spectrum estimate as:

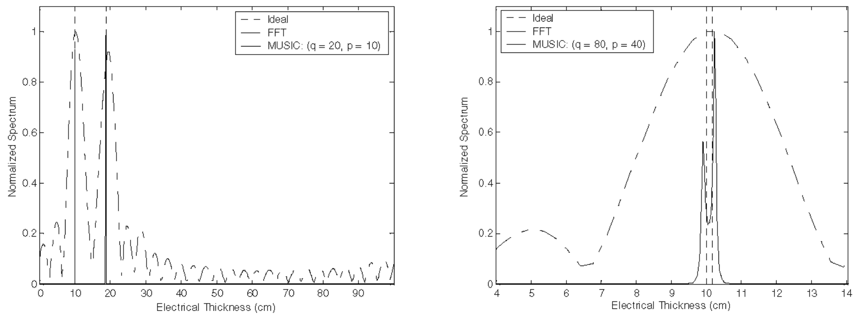
$$S(d) = \frac{1}{\mathbf{a}_{\mathbf{s}}(d)\mathbf{V}_w\mathbf{V}_w^H\mathbf{a}_{\mathbf{s}}^H(d)} \tag{7}$$

where $\mathbf{a}_s(d) = \begin{bmatrix} e^{-2j\beta(f_1)d} & e^{-2j\beta(f_2)d} & \cdots & e^{-2j\beta(f_p)d} \end{bmatrix}^T$ is the sub-measurement delay scanning vector, and the delay scanning range is $0 < d < \frac{c}{2\delta_f}$.

NUMERICAL RESULTS

The simulations were intended to introduce and present the relative performance of the proposed technique for dielectric slab thickness measurement scenario. To this end, the normalized delay spectrum as estimated from the MUSIC algorithm and FFT-based was investigated for different slab thicknesses. Although the absolute spectrum value at a certain delay is generally needed (i.e., in inverse model), we are mainly interested in relative delay spectrum. In other words, it is sufficient for the application at hand to locate two peaks on the spectrum (detection) and measure the delay between them (delay resolution).

The frequency response of a lossless dielectric slab of varying (physical) thickness d_s, and $\varepsilon_r' = 3$ was simulated in the X-band frequency range ($8.2 - 12.4$ GHz).



(a) **(b)**

FIGURE 2: The MUSIC and FFT delay spectrums for slab of thickness (a) 5 cm and (b) 0.1 cm.

The slab is assumed to be positioned at $d_o = 10$ cm away from the illuminating antenna. For the FFT algorithm, the thickness resolution is 2.06 cm.

FIGURE 2(a) shows the normalized delay spectrum as computed from the FFT algorithm and the MUSIC algorithm for when the slab thickness is set to $d_s = 5$ cm using the entire X-band bandwidth (4.2 GHz). Ideally, we would like to see two fine spectral lines (corresponding to the slab's interfaces) one located at $d_o = 10$ cm and the other at $d_o + \sqrt{3}d_s = 18.7$ cm. As shown in FIGURE 2(a), both algorithms detected the slab interfaces and provide accurate estimation of the slab thickness.

To challenge the resolution of the FFT algorithm, the slab physical thickness was reduced to $d_s = 0.1$ cm while keeping the other parameters fixed (in this case, $d_o + \sqrt{3}d_s = 10.173$ cm). FIGURE 2(b) shows the normalized delay spectrum as computed by the FFT and the MUSIC algorithms. As expected, the FFT algorithm failed to resolve both interfaces. On the other hand, the MUSIC algorithm detected and located both interfaces accurately. This example shows that the resolution provided by the MUSIC algorithm is at least 20 times finer than the one obtained by the FFT algorithm. This improvement translates in a significant reduction in the complexity and cost of the inspection system.

EXPERIMENTAL RESULTS

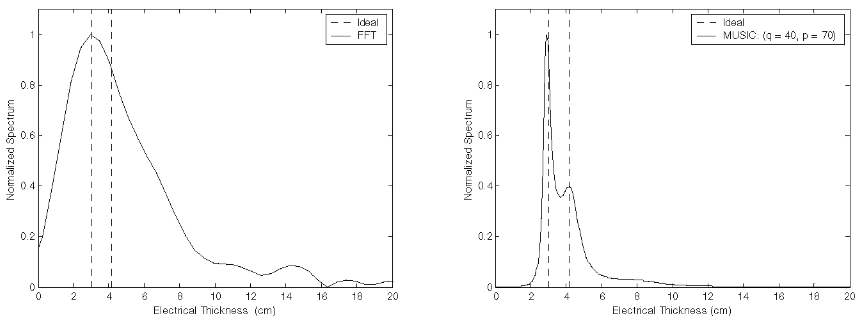
Although the results presented here are drawn from controlled laboratory experiments, the used experimental setup was designed such that it mimics, as closely as possible, the practical testing environment.

Thickness estimation of a synthetic rubber and Plexiglass slabs irradiated by a wideband microwave signal (Ku-band: $12.4 - 18$ GHz) in the far-field of an open-ended rectangular waveguide (not very directive antenna) was considered. The measured relative permittivities of the rubber and Plexiglass were 7.28-j 0.27 and 2.62-j0.015, respectively. The nominal slab thickness was measured to be 0.442 cm for the rubber slab and 1.14 cm for the Plexiglass slab. Finally, for both slabs, the waveguide aperture was placed at $d_o = 3$ cm away from the first slab-air interface.

The frequency response, i.e., the complex reflection coefficient, of each slab was measured using an HP8510C vector network analyzer. Free-space measurements, where the waveguide is directed toward an free-space, is used to set the measurement reference for the internal device reflections.

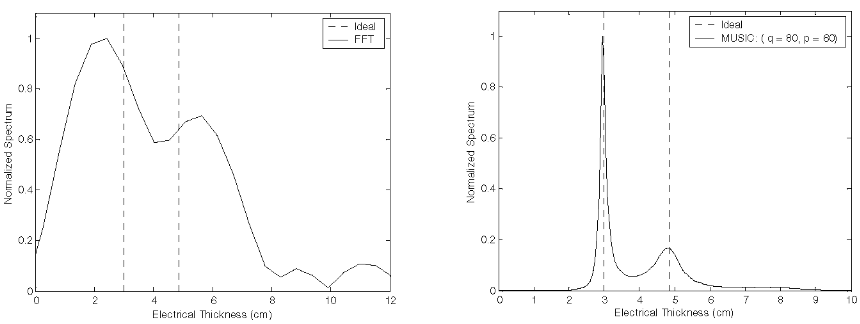
FIGURE 3(a) shows the normalized delay spectrum as obtained using the FFT algorithm on the rubber slab response. It is evident that FFT algorithm missed the second intrinsic reflection, and consequently, was not able to estimate the thickness of the slab. On the other hand, FIGURE 3(b) shows that MUSIC algorithm detected both interfaces and estimated the thickness as 0.463 cm (4.75% error).

For the Plexiglass slab, the FFT algorithm was able to detect both interfaces and estimated the thickness as 1.988 cm (74.21% error) as indicated in FIGURE 4(a). The MUSIC delay spectrum as depicted in FIGURE 4(b) indicates the presence and accurate location of the interfaces. The estimated Plexiglass slab thickness using the MUSIC algorithm was 1.143 cm (0.26% error).



(a) **(b)**

FIGURE 3: The delay spectrums for a 0.442 cm Rubber slab using (a) FFT and (b) MUSIC algorithms.



(a) **(b)**

FIGURE 4: The delay spectrums for a 1.14 cm Plexiglass slab using (a) FFT and (b) MUSIC algorithms.

SUMMARY AND DISCUSSION

Remote dielectric slab thickness measurement is frequently required in many industrial applications. Illuminating the slab with wideband microwave signals and analyzing the delay spectrum of the reflected signal can address these applications effectively. In this paper, the MUSIC spectrum estimator was applied to the frequency swept measurements of the reflection coefficient for irradiated slabs. It was shown that the MUSIC algorithm provides higher thickness resolution and requires less bandwidth compared to the FFT based techniques. Consequently, the proposed technique is more feasible for applications where the considered slabs are relatively thin.

The simulations conducted in this investigation showed that the MUSIC algorithm needs only a small fraction (~5%) of the bandwidth required by the FFT algorithms to provide adequate resolution. The efficacy of the proposed approach was also demonstrated experimentally by considering two different slabs made of high-loss and low-loss materials. In both cases, the MUSIC algorithm provided accurate thickness measurements.

Applying the MUSIC algorithm requires dealing with the coherence of the measured signal effectively. For this purpose, spatial smoothing was utilized where the smoothing parameters need to be optimized before application.

REFERENCES

1. Zoughi, R., *Microwave Non-Destructive Testing and Evaluation*, Kluwer Academic Publishers, the Netherlands, 2000.
2. D. B. Percival and A. T. Walden, *Spectral Analysis for Physical Applications*, Cambridge University Press, UK, 1993.
3. C. A. Balanis, *Advanced Engineering Electromagnetics*, New York, John Wiley & Sons, INC., 1989.
4. R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Anten. and Propag.*, vol. AP-34, No. 3, pp. 276-280, March 1986.
5. T-J. Shan, M. Wax, and T. Kailath, "On spatial smoothing for direction-of-arrival estimation of coherent signals," *IEEE Trans. Acous., speech, and Sign. Process*, vol. ASSP-33, No. 4, pp. 806-811, August 1985.
6. S. U. Pillai, and B. H. Kwon, "Forward/backward spatial smoothing techniques for coherent signal identification," *IEEE Trans. Acous., speech, and Sign. Process.*, vol. 37, No. 1, pp. 8-15, Jan. 1989.
7. M. Wax and T. Kailath, "Detection of signals by information theoretic criteria," *IEEE Trans. Acoust. Speech, Signal Processing*, vol. 33, Feb. 1985.